

EVALUATION OF HUMAN EXPOSURE TO SINGLE ELECTROMAGNETIC PULSES OF ARBITRARY SHAPE

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SUMMARY

Transient current density $J(t)$ induced in the body of a person exposed to a single magnetic pulse of arbitrary shape or to a magnetic jump is filtered by a convolution integral containing in its kernel the frequency and phase dependence of the basic limit value adopted in a way similar to that used for reference values in the International Commission on Non-Ionising Radiation Protection statement. From the obtained time-dependent dimensionless impact function $W_J(t)$ can immediately be determined whether the exposure to the analysed single event complies with the basic limit. For very slowly varying field, the integral kernel is extended to include the softened ICNIRP basic limit for frequencies lower than 4 Hz.

Key words: exposure limits, exposure evaluation, pulsed magnetic field, pulsed body currents, non-ionizing radiation, low frequency field

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INTRODUCTION

The health risk coming from the exposure to low frequency electric and magnetic fields began to be systematically investigated in the last two decades. Consistent guidelines for limiting exposure comprising the whole low-frequency interval from zero frequency (static magnetic and electric fields) to several megahertz were issued in 1998 by ICNIRP (International Commission on Non-Ionising Radiation Protection) (1). The basic limits for the human exposure have been set substantially lower than the well-established effect of stimulation of nerves by the induced electric currents. Electrolytic decomposition of the body liquids or burn of the tissue would appear at much higher currents. So the compliance with the limits excludes e.g. any disturbance of the physiological currents controlling the beating of heart or processes in the brain.

In workplaces, magnetic fields able to induce the body currents exceeding the limits are not exceptional. They are found not only in institutions using very strong electric currents for inductive heating or for electric welding and similar procedures. Devices for magnetotherapy, antitheft gates, appliances for pulsed magnetization of electronic components and other low-power sources of varying magnetic fields may sometimes also generate fields violating the limits.

In recent statement of the ICNIRP (2) was shown how to evaluate human exposure to strongly non-sinusoidal and pulse-shaped electromagnetic fields. To that purpose, ICNIRP slightly changed the frequency dependence of reference values set in (1), and used the sum of weighted Fourier components of measured magnetic flux density $B(t)$ or of its time derivative $dB(t)/dt$ to check the compliance with reference values set in (1). Detailed justification of the procedure was published in (3).

In order to compare the results of measurements of single magnetic pulses or jumps with ICNIRP guidelines, it may be more convenient to use direct weighing of the ICNIRP basic limits (i.e. of current density induced in the body) by integral convolution yielding a time-dependent dimensionless function

$W_J(t)$ which will be called impact function, rather than manipulating with the (continuous) Fourier spectrum. To that purpose, the ICNIRP piecewise straight log-log frequency dependence of current density limit was smoothed near the frequency 1 kHz. In order to include also very slowly varying transients, the softening of current density limit below 4 Hz has been included into the convolution kernel.

Basic limit values for occupational exposure are used throughout the paper. For general public the basic limit values are five times lower.

FORMULATION OF THE PROBLEM

Let

$$J(t, x) = \text{Const} \frac{dB}{dt} \quad [1]$$

be the analysed electric current density induced in the body, with *Const* depending on the specific electric conductivity of the tissue and on the area and length of the loop closing the induced current. At frequencies lower than, say, 1 MHz, the time-varying magnetic field $B(t)$ is the main source of this current, and following considerations are confined to magnetic field only. To find out compliance with basic limits, the current density $J(t)$ will be filtered by a convolution integral

$$W_J(t) = \int_{-\infty}^{\infty} g(t - \tau) J(\tau) d\tau \quad [2]$$

accounting for the physiological effectiveness of frequencies contained in the time-varying current. Convolution kernel $g(t)$ is chosen so that the integral gives dimensionless function $W_J(t)$, absolute value of which is at any time smaller or equal to 1, if the induced body current density $J(t)$ does not exceed the ICNIRP current density limit.

CONSTRUCTION OF CONVOLUTION KERNELS

The best fit for smoothed frequency dependence for inverted current density limit function analogical to that used for $dB(t)/dt$ in (2) is

$$G_{fast}(\omega) = \frac{Q}{1 + j\omega RC} \quad [3]$$

with $Q=100/\sqrt{2} \text{ m}^2\text{A}^{-1}$ as multiplication factor, ω as angular frequency, $RC=1/(2\pi f_{corner})$ as time constant and with $f_{corner}=1000 \text{ Hz}$ as at the “corner” frequency of $J(f)$ limit values introduced in (1).

The corresponding integral kernel $g(t)$ is

$$g_{fast}(t) = \frac{Q}{RC} e^{-\frac{t}{RC}} H(t) \quad [4]$$

$H(t)$ is the Heaviside unit step function.

The relevant equivalent circuit is the same as that used in (2) for weighing $dB(t)/dt$ by reference values and is shown in Fig. 1. Multiplication factor Q is implemented as direct amplification and time constant RC is implemented as connection of resistor R and capacitor C . Since only the time constant, term RC , is of importance, resistor or capacitor can be chosen arbitrarily.

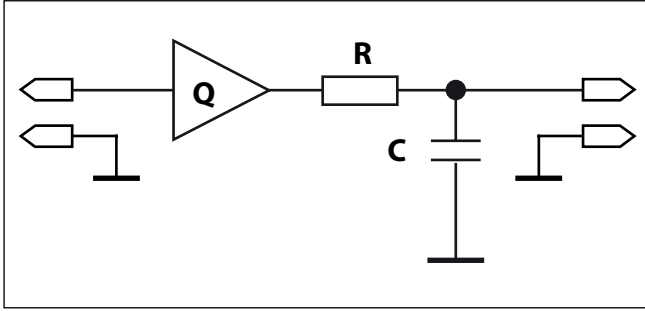


Fig. 1. Equivalent circuit for fast integral kernel.

In order to include the softening of current density limit between 1 Hz and 4 Hz, convolution kernel

$$g_{slow}(t) = Q \left[\delta(t) - \frac{R_1}{L} e^{-\frac{t(R_1+R_2)}{L}} H(t) \right] \quad [5]$$

is used, with L chosen equal to one and $R_2=R_1/3$. $R_1/L=21$.

The corresponding complex frequency dependence is

$$G_{slow}(\omega) = Q \frac{R_2 + j\omega L}{R_1 + R_2 + j\omega L}, \quad [6]$$

with equivalent circuit shown in Fig. 2. Here the parameters L and R_1, R_2 have dimensions of inductance and resistance and can be thus implemented in this way.

Combining both the slow and the fast convolution kernels gives the full convolution kernel

$$g_{full}(t) = Q \frac{(RCR_2 - L)e^{-\frac{t}{RC}} + R_1 RC e^{-\frac{t(R_1+R_2)}{L}}}{RC(RC R_1 + RCR_2 - L)} H(t) \quad [7]$$

with the corresponding complex frequency dependence

$$G_{full}(\omega) = Q \frac{R_2 + j\omega L}{R_1 + R_2 + j\omega L} \cdot \frac{1}{1 + j\omega RC} \quad [8]$$

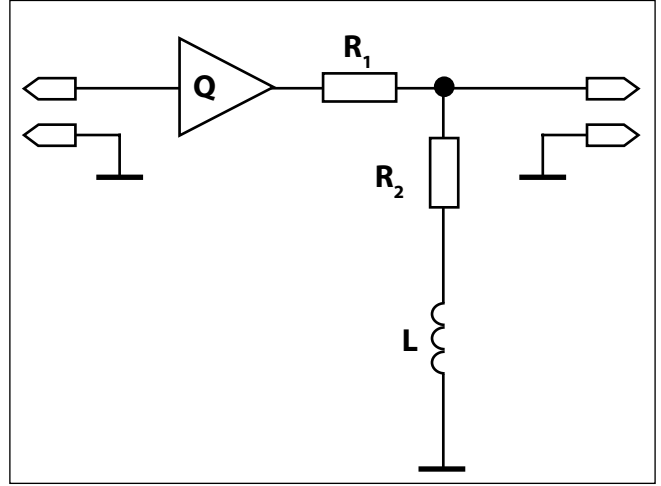


Fig. 2. Equivalent circuit for slow integral kernel.

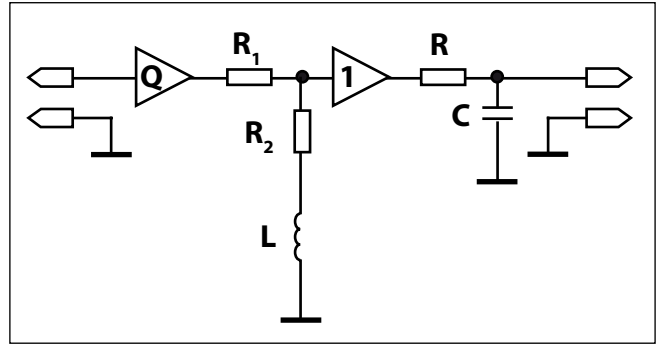


Fig. 3. Equivalent circuit for full integral kernel.

and equivalent circuit shown in Fig. 3. All circuit parameters have the same meaning as was stated above.

In Fig. 4 the log-log plots of the transient response

$$h(t) = \int_0^t g(z) dz \quad [9]$$

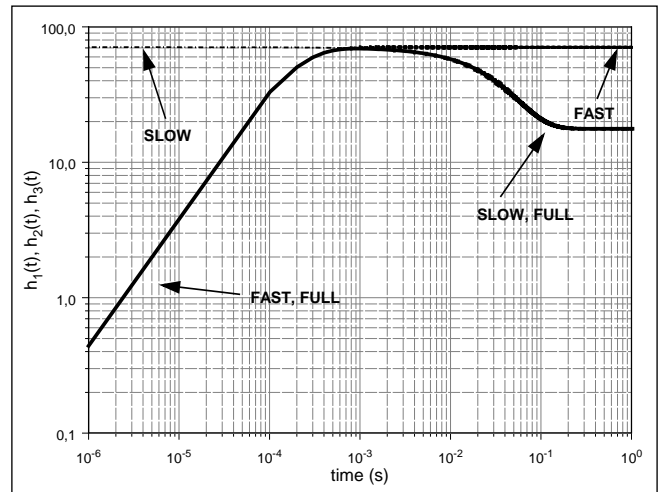


Fig. 4. Transient response of equivalent circuit for slow, fast and full integral kernel to Heaviside unit step.

to Heaviside unit step are shown with each equivalent circuit along.

Time-span between the two parts of g_{full} is too large to allow flawless numerical computation using the full form of the kernel, and g_{slow} must usually be used separately for numerical calculations of low-frequency signals with sharply changing parts.

ANALYTICAL EXAMPLE

A pulse $I(t)$ of electric current flowing in the conductive channel of a lightning can be approximated by

$$I(t) = 15 \cdot 10^3 [1 + \text{erf}(10^{6.5} t - 2)] e^{-10^{4.2} t} \quad [10]$$

Induced current density in the body of exposed person (for a straight, long current channel) at a distance of 1 m ($J, r=1\text{m}$) can be calculated with the help of Faraday Law and equation [1] with $Const=0.022$ siemens, chosen for the largest body loop. Its time course is depicted in Fig. 5. The time interval of non-zero values of $J(t)$ is infinite in this case, but for $t \leq 0$, its values are negligible in comparison to the peak value.

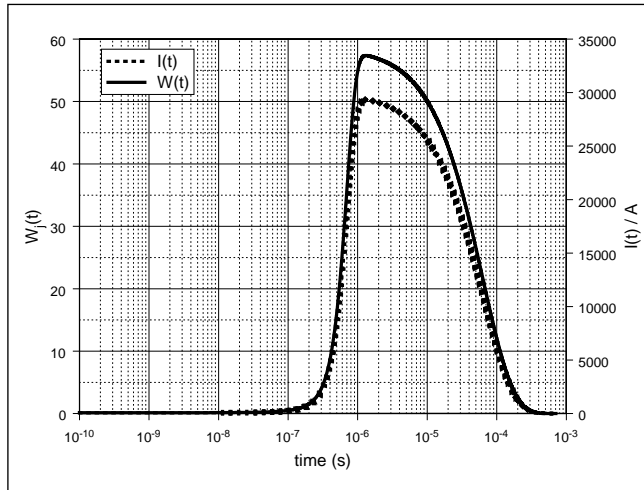


Fig.5. Time course of impact function for single pulse of electric current flowing in the conductive channel of a lightning in the distance of one meter.

Fig. 5 also depicts the time course of

$$W_j(t) = \int_{-\infty}^{\infty} g_{full}(t - \tau) J(\tau, r = 1\text{m}) d\tau \quad [11]$$

The ICNIRP current density limit has now been exceeded by a factor approximately equal to 60.

The result corresponds to a typical current value (4) in lightning conductive channel, while even four times higher values of current have been reported. So, in extreme cases, $W_j(t)$ could reach a value of 200 or more at a one-meter distance from the lightning, while the ICNIRP basic limit would be satisfied only in a distance of 200 meter and 1000 meter for occupational and general public exposure, respectively. Though lightning with the highest estimated current may appear only rarely, it can be inferred from quite a large number of lightning strokes appearing on the earth, that the ICNIRP limits are safe indeed: deaths suspected to

be caused by lightning which did not hit a person by direct impact have still not been with certainty established.

EXAMPLES OF MEASURED FIELDS

Weighing the time dependent field in the time domain can be used for periodic signals, too. Fig. 6 shows the magnetic flux density obtained from an oscilloscope record with the probe located at the top edge of an antitheft device.

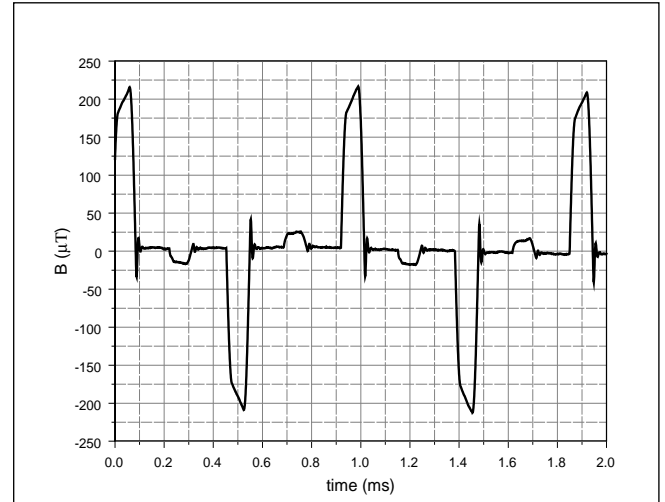


Fig. 6. Magnetic flux density obtained from an antitheft device.

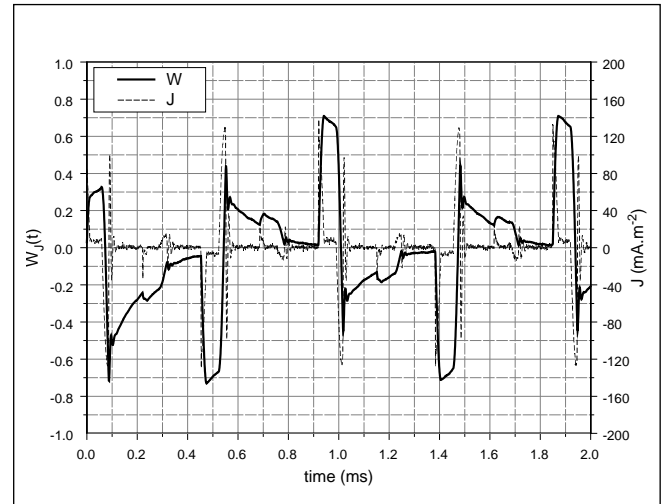


Fig. 7. Induced body current density $J(t)$ and the computed impact function $W_j(t)$ for magnetic flux density obtained from the antitheft device.

The signal is periodic, and weighing the components of its discrete Fourier spectrum would be the standard evaluation procedure. Nevertheless, the direct filtering by the convolution described above can be used, too. Fig. 7 shows two periods of the induced body current density $J(t)$ and the computed impact function $W_j(t)$, both calculated from the measured $dB(t)/dt$ curve.

The transient part at the beginning influences the time course of the impact function, while the steady part of $W_j(t)$ can be used

for comparison of the exposure with the basic limit value. Value of the *Const* was chosen 0.01 siemens in this case, which corresponds to the size of the current loop in the head of exposed person.

NOTE ON SIGNAL CORRECTION

The time-course of $dB(t)/dt$ is only approximately proportional to the time-varying voltage $U(t)$ on the induction coil usually used for precise measurement. If the signal contains frequencies influenced by the coil self inductance, the equivalent circuit (see Fig. 8) of coil and oscilloscope input port must be taken into account.

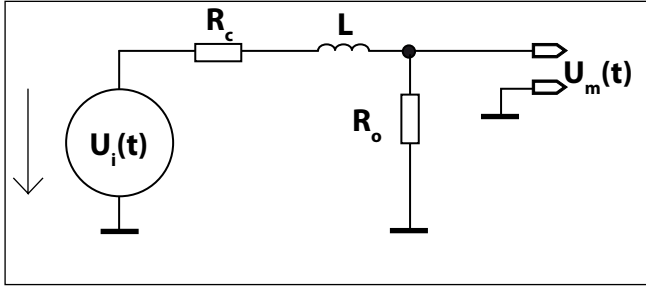


Fig. 8. The equivalent circuit of measuring coil connected to oscilloscope input port.

If

$$u_i(t) = S \frac{dB(t)}{dt} \quad [12]$$

then

$$\frac{dB(t)}{dt} = \frac{L_c}{S_c R_o} \frac{du_m(t)}{dt} + \frac{1}{S_c} \left(1 + \frac{R_c}{R_o} \right) u_m(t) \quad [13]$$

where $U_m(t)$ is the voltage recorded by oscilloscope, L_c, R_c are self inductance and resistance of the coil, respectively, R_o is the oscilloscope port resistance and S_c the effective area of coil.

The calibrated coil used for oscillographic records used here had 36 turns, $L=415 \mu\text{H}$, $R_c=10 \Omega$, $S_c=0.5 \text{ m}^2$, and $R_o=50 \Omega$. The correction was significant at frequencies beyond 10 kHz. The described correction does not take into account the capacity of the coil and the capacity of the oscilloscope input. Their influence would become important at frequencies higher than 1 MHz, for which it would be more reasonable to measure with another coil.

The correction can be made part of the convolution kernel.

NOTE ON NUMERICAL EVALUATION OF IMPACT FUNCTION

It is known that numerical evaluation of convolution integral is time consuming operation. However it will be shown that for the case of integral kernels used here, the evaluation time can be reduced dramatically. To this point assume the convolution in the form

$$S(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau \quad [14]$$

where $g(t)$ and $f(t)$ are kernel and filtered function respectively. Without loss of generality $f(t)=0$ for $t<0$ can be assumed as well as the causality of the kernel: $g(t)=0$ for $t<0$. Equation (14) is then

$$S(t) = \int_0^t f(\tau) g(t-\tau) d\tau \quad [15]$$

where $S(t)=0$ for $t<0$. Assume now the integral kernel in the form

$$g(t) = \beta e^{-\alpha t} H(t) \quad [16]$$

from which all kernels used here can be constructed.

For the case of data obtained by a measurement in the case of numerical evaluation, the continuous functions are then sampled with the equidistant time step Δt . Then following relation can be deduced

$$S(t + \Delta t) = e^{-\alpha \Delta t} S(t) + \beta e^{-\alpha \Delta t} \int_t^{t+\Delta t} f(\tau) e^{-\alpha(t-\tau)} d\tau \quad [17]$$

Last integral can be in real situations for sufficiently small Δt approximated by

$$\int_t^{t+\Delta t} f(\tau) e^{-\alpha(t-\tau)} d\tau \approx \frac{\Delta t}{2} [f(t) e^{-\alpha(t-t)} + f(t+\Delta t) e^{-\alpha(t-t+\Delta t)}] \quad [18]$$

Finally approximate value of convolution integral in next time step can be computed as

$$S(t + \Delta t) \approx e^{-\alpha \Delta t} S(t) + \frac{\beta \Delta t}{2} [f(t) e^{-\alpha t} + f(t + \Delta t) e^{-\alpha(t+\Delta t)}] \quad [19]$$

from already known values of α , β , $f(t)$, $S(t)$. This gives a fast recurrent formula for numerical evaluation of [15] since $S(0)=0$.

DISCUSSION

While single events are processed by convolution integral exactly, the impact function $W_j(t)$ of measured periodic signals, recorded necessarily as finite in time, has a transient part at the beginning. If the time series of measured signal is longer than the transient, and the rest of the signal contains at least one period, then the transient part can be thrown away and the steady part can be used in the same way as in case of nonperiodic solitary events: if $\max |W_j(t)| \leq 1$, the basic limit is not exceeded.

Evaluating periodic signals, it is necessary to be sure that the value of the integral over the period is zero, as long as DC bias sometimes appears on digitally recorded signals. Such a spurious shift would lead to incorrect values of the impact function and hence to inaccuracies in the exposure evaluation. With single events (solitary pulses) this danger does not exist.

CONCLUSION

The presented method of evaluation of exposure deals directly with the basic limits set for induced body current, which allows correct evaluation of exposure while taking into account the

inhomogeneity of the field and the only exposed part of the body by choosing appropriate value for $Const$ in eq [1].

The convolution accounting for physiological effectiveness of the induced body current at different frequencies is most convenient for single events, e.g. for isolated pulses or jumps. Nevertheless, the method can also be used for periodic signals of any form, if the transient process at the beginning can be ignored and only the steady (periodic) part of the impact function $W_f(t)$ is used for evaluation.

Another advantage of convolution integral is that it excludes spurious oscillations at the beginning and end of abrupt jumps in recorded signal appearing due to finite number of harmonics used in the Fourier series and due to noncausality of the Fourier transformation.

As long as the slow response has been taken into account in the convolution kernel, less stringent results are obtained for the exposure to very slowly varying signals.

Only simple form of the convolution kernel was used in this paper. The applied method allows to construct kernels of more complicated analytical form or kernels defined numerically.

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